ANALYSIS OF MICROWAVE FERRITE DEVICES BY THE TRANSFINITE ELEMENT METHOD

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ABSTRACT

The transfinite element method is used to model microwave ferrite devices. It is shown that this procedure is both easy to set up and inexpensive to solve. A computer implementation is described that gives the fields and scattering parameters in microwave ferrite devices of arbitrary two-dimensional geometry.

INTRODUCTION

Ferrite components such as isolators and circulators are often used in microwave systems. With such devices, analytic and semi-analytic methods are of limited value due to the complexity of the electromagnetic fields. Consequently, numerical methods must be employed to provide general analysis capabilities. Unfortunately, however, the majority of existing numerical design tools are based on integral equation methods that require special treatment across material boundaries, neglect the off-diagonal terms in the permeability tensor, or assume no ferrite losses [1] [2]. A more realistic procedure for modeling microwave ferrite devices was recently developed by Koshiba and Suzuki [3], who used the finite element method together with Green's functions at the microwave ports to solve the ferrite device field problem.

This paper presents a new procedure for modeling microwave ferrite devices. The new procedure is based on the transfinite element method that employs analytic basis functions to approximate the fields extending to infinity at the microwave ports. This procedure is considerably easier and more efficient than the one presented in [3] because the boundary conditions at the ports are satisfied automatically by the variational method along a single reference plane at each port. With the method presented in [3], boundary conditions at ports must be imposed explicitly by the use of Green's functions along two planes in each port.

FORMULATION

The application of the transfinite element method to isotropic microwave circuit devices is reported in a companion paper [4]. Here we present the extension of the procedure to microwave ferrite devices.

The permeability tensor $[u]$ of a general ferrite material is given in reference [5] as

$$[u] = u_0 \begin{bmatrix} u_r & -j\kappa_r & 0 \\ j\kappa_r & u_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

where

$$u_r = 1 + \frac{(\omega_0 + j\omega\alpha)\omega_m}{(\omega_0 + j\omega\alpha)^2 - \omega^2}$$

$$\kappa_r = -\frac{\omega\omega_m}{(\omega_0 + j\omega\alpha)^2 - \omega^2}$$

Applying Galerkin's method to the wave equation, we are led to the following bilinear functional

$$B(\phi,\psi) = \int_\Omega \frac{u_r}{u_r^2 - \kappa_r^2}\left(\frac{\partial\phi}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\psi}{\partial y}\right) d\Omega +$$

$$\int_\Omega \frac{j\kappa_r}{u_r^2 - \kappa_r^2}\left(\frac{\partial\phi}{\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial\phi}{\partial y}\frac{\partial\psi}{\partial x}\right) d\Omega - k_0^2 \int_\Omega \epsilon_r \phi\psi \, d\Omega - \int_\Gamma \phi \frac{\partial\psi}{\partial n} d\Gamma \qquad (2)$$

As in [4], analytic basis functions are used to approximate the fields outside the ports and finite elements are used in the discontinuity region. Imposing continuity of the field of the finite element nodes along the port boundaries results in the following matrix equation

$$[C]^t \sum_{\Omega_e} \left(\frac{u_r}{u_r^2 - \kappa_r^2}[S] + \frac{j\kappa_r}{u_r^2 - \kappa_r^2}[M] - k_0^2[T]\right)[C]\hat{\psi} = \hat{y} \quad (3)$$

The connection matrix $[C]$ in (3) is almost an identity matrix; hence, the above transformation can be performed very efficiently.

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THE ELEMENT MATRIX $[M]$

Triangular finite elements are often used in electrical engineering because these elements can be integrated analytically and tabulated for any triangle shape. The element matrices $[S]$ and $[T]$ in (3) are common to isotropic electromagnetics problems and are given in [6]. The matrix $[M]$, on the other hand, is new. It is given by

$$[M]_{mn} = \int \left(\frac{\partial \alpha_m}{\partial x}\frac{\partial \alpha_n}{\partial y} - \frac{\partial \alpha_m}{\partial y}\frac{\partial \alpha_n}{\partial x} \right) d\Omega \tag{4}$$

where α_i; are the usual Lagrange finite element shape functions [6]. Notice that the matrix $[M]$ is anti-symmetric. Using simplex coordinates, the integral in (4) is written as

$$[M]_{mn} = \sum_{i=1}^{3} \int \frac{\partial \alpha_m}{\partial \zeta_i}\frac{\partial \alpha_n}{\partial \zeta_{i+1}} - \frac{\partial \alpha_m}{\partial \zeta_{i+1}}\frac{\partial \alpha_n}{\partial \zeta_i} d\Omega \tag{5}$$

Evaluating this for first and second order triangles gives

$$M^{(1)} = \frac{1}{2}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$M^{(2)} = \frac{1}{6}\begin{bmatrix} 0 & -4 & 4 & 1 & 0 & -1 \\ 4 & 0 & 0 & -4 & 0 & 0 \\ -4 & 0 & 0 & 0 & 0 & 4 \\ -1 & 4 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 4 & 0 & -4 \\ 1 & 0 & -4 & -1 & 4 & 0 \end{bmatrix} \tag{6}$$

Results

Figure 1 shows the real and imaginary parts of the contours of constant E_z field for a Y-junction circulator with a central ferrite post (TT1-109). At the circulation frequency, the transfinite element method predicts the following:

$$ReflectionLoss = -16.5(dB)$$
$$IsolationLoss = 19.0(dB)$$
$$IsertionLoss = 0.5(dB)$$

The values agree with the results in [3] to within the accuracy of the published graphs. A second example is presented in Figure 2. This figure shows the lines of constant E_z computed by the transfinite element method for a planar ferrite circulator on a GaAs substrate at a frequency of 11 GHz.

References

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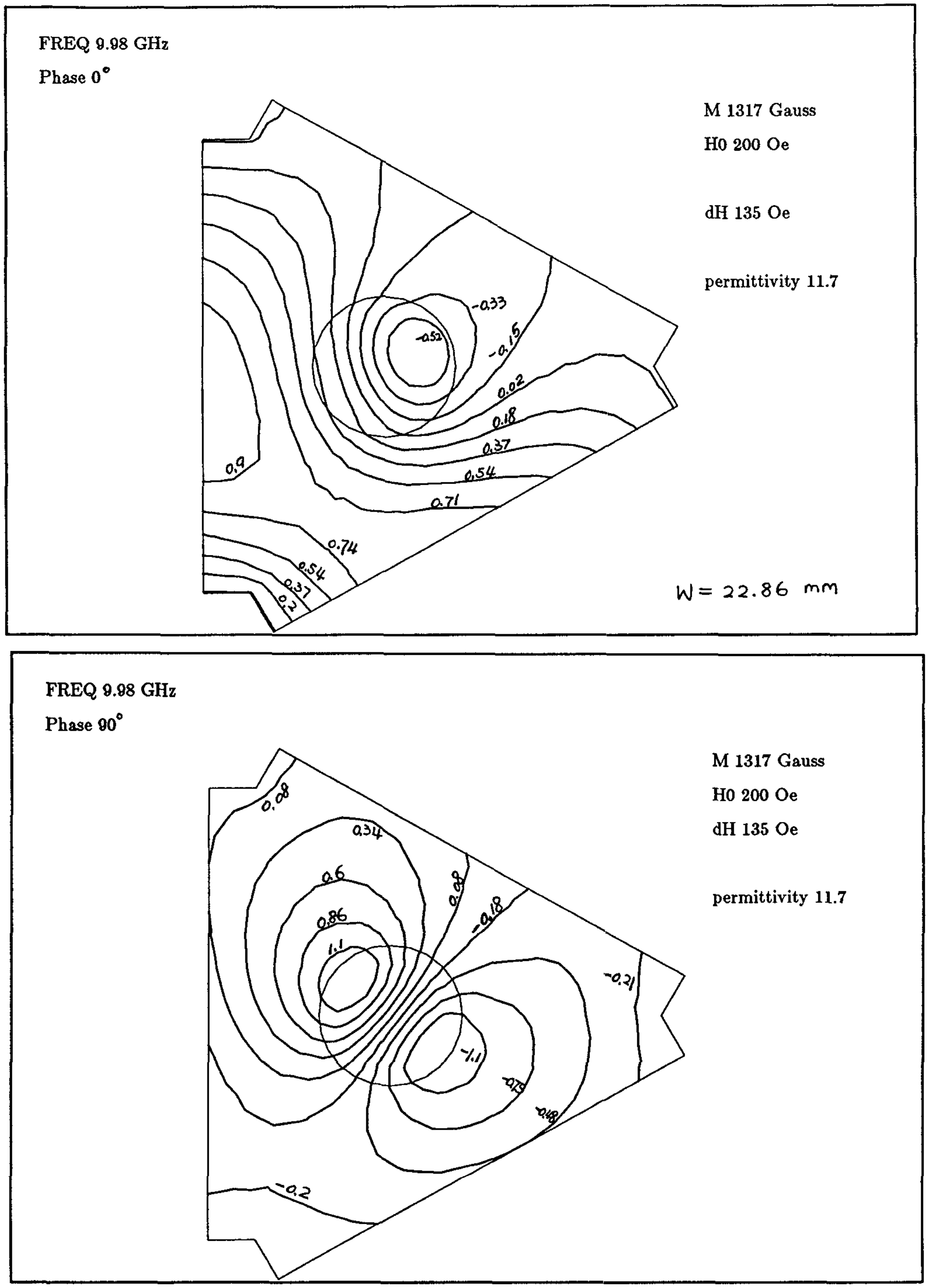

Figure 1: Contours of constant E_z in a waveguide Y-junction circulator containing a ferrite post at 9.98 GHz. (a) time phase 0°, (b) time phase 90°.

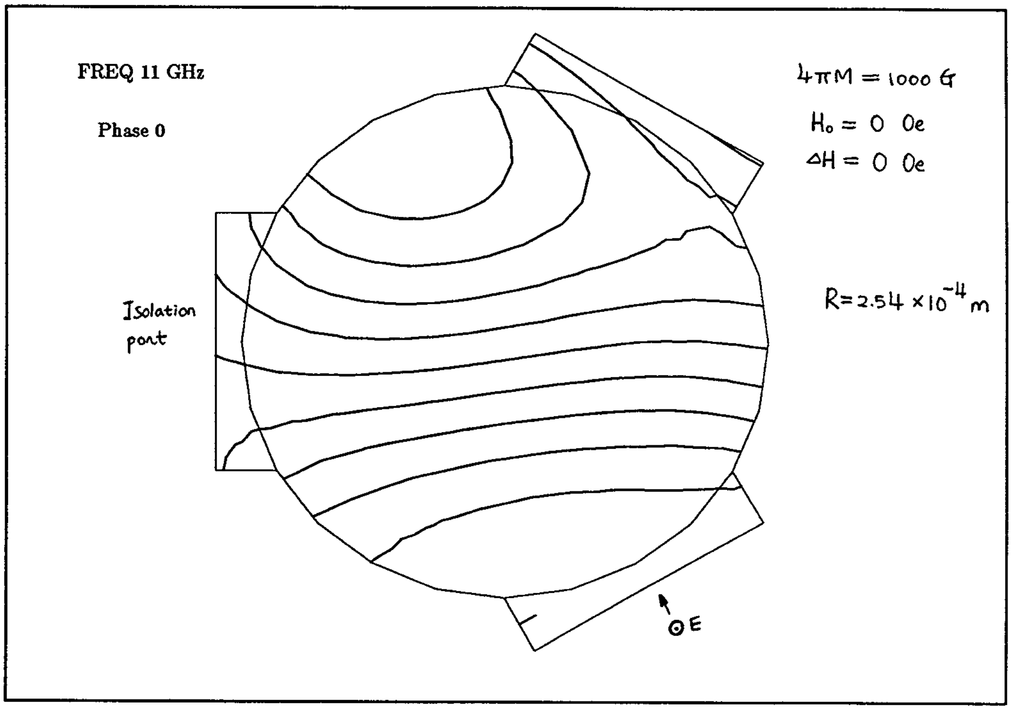

Figure 2: Contours of constant E_z in a planar ferrite circulator on a GaAs substrate 11 GHz and time phase 0^o.